

Sigma/Glueball Decay of D^+ and D_s^+

Leonard S. Kisslinger[†]

Department of Physics, University of Maryland, College Park, MD 20742

February 7, 2008

Abstract

Recently the D^+ charm meson was observed to have a clear branching ratio into the low energy $\pi - \pi$ sigma resonance, while this channel was not detected in the D_s^+ decay. It is shown that this is consistent with the standard treatment of exclusive charm meson decays and a proposed glueball/sigma picture.

PACS Indices:13.25.Ft,14.40.Cs,12.38.Lg,14.40.-n,14.70.Dj,13.75.Lb

The Fermilab E791 Collaboration has recently reported seeing a clear signal of the decay of the D^+ into a π^+ plus a low-energy scalar-isoscalar $\pi - \pi$ resonance, with a (mass,width) of about (480,325) MeV[1]. This is presumably the elastic resonance found[2] in the analysis of $\pi - \pi$ scattering with (mass,width) of about (400,400) MeV. We refer to this as the sigma, σ . In the D_s^+ decays, however, the sigma was not found[3]. It is the purpose of the present work to show that these experimental results are consistent with the glueball/sigma model which we have recently proposed. With this model there is a unique skeletal diagram which allows us to calculate the ratio of the $\sigma\pi^+$ to the $\phi\pi^+$ decay widths of the D^+ with no new parameters. The QCD corrections to the skeletal diagrams for these decay channels are then discussed. For the D_s^+ decay into the $\sigma\pi^+$ channel it is shown that there are two skeletal diagrams then tend to cancel, which can explain why this channel has not been observed in experiments.

The sigma glueball idea is based in part on the QCD sum rule analysis of the scalar gluon correlator and in part from the large σ decay rates of scalar glueball candidates. Although the QCD sum rule method is not used in the present work, we briefly review the method for finding glueball masses. Using the composite field operator for the scalar

glueball, $J^G(x) = \alpha_s G^2$, where $G_{\mu\nu}^a$ is the gluon field tensor and $J^G(x)$ is proportional to the pure-gluon term in the QCD Lagrangian, the scalar gluon correlator is defined as

$$\Pi(p) = i \int d^4x e^{iq \cdot x} \langle 0 | T[J(x)J(0)] | 0 \rangle, \quad (1)$$

where the link operator has been omitted. The correlator is evaluated numerically in lattice gauge calculations. In the QCD sum rule method the dispersion relation for the correlator is equated to a QCD operator product expansion, with the mass of the Glueball determined by estimating the value of the pole in the dispersion relation. Using a subtracted dispersion relation and carrying out the standard QCD sum rule analysis[4] a number of theorists find a glueball solution in the region of 300-600 MeV[5], depending on the values used for the nonperturbative condensate parameters. In a recent study of the coupled scalar mesons and glueballs in which instanton as well as other gluonic effects have been included[6] we find a mainly meson solution corresponding to the $f_0(1370)$, a mainly glueball solution corresponding to the $f_0(1500)$, and a light glueball in the region of 400-500 MeV[6], consistent with our earlier studies. The main two sources of error in the method are the determination of the values of the condensates and the treatment of the continuum part of the dispersion relation. A light glueball has not been found in quenched lattice gauge calculations.

The other observations which lead to the glueball/sigma picture are first that the $f_0(1500)$, which has characteristics of a glueball with a scalar meson admixture, has the four-pion channel as its largest decay branching fraction[7]; and second, that a BES analysis[8] has shown that the four- π channel is dominated by two-sigmas. This is also true for the $f_0(1710)$ and $f_0(2100)$ which have recently been shown to have glueball characteristics. This has led to our conjecture that the light glueball and sigma form a coupled-channel system, with the sigma resonance driven by the glueball pole. From the Breit-Wigner resonance fit to the resonance[2] we obtain the matrix element $\langle \sigma | V^{int} | GB \rangle$ for the coupling of the scalar glueball to the scalar $\pi - \pi$ resonance. Note that the coupling interaction V^{int} is not needed in this work, only the matrix element taken from the experimental analysis. We have used this for the study of hybrid baryon decay[9], and the pomeron-nucleon coupling and the production of sigmas in high energy p-p collisions via the pomeron[10]. This is the glueball/sigma model. Since it is possible that the treatment of the gluonic continuum could give a false solution for the light glueball, we consider this picture to be a conjecture, however, the model would also follow from a very broad gluonic structure in the region of the σ (i.e., the glueball pole is far from the real axis) which couples strongly to the sigma resonance. If the picture is valid there are many important consequences that can be observed in a variety of experiments. We believe that the charm meson decays are an excellent example, which is the subject of the present work.

It is convenient to use the quark-diagram classification scheme[11], in which there are six skeletal diagrams, by which we mean processes without explicit gluonic effects. The need to consider processes in addition to what was once considered to be the dominant spectator decay process was made evident by the large difference between D^+ and D^0 lifetimes[12]. See Ref [13] for a review of exclusive D^+ , D^0 and D_s^+ charm decays expressed in terms of this classification. Theoretical treatments have been mainly based on an effective weak

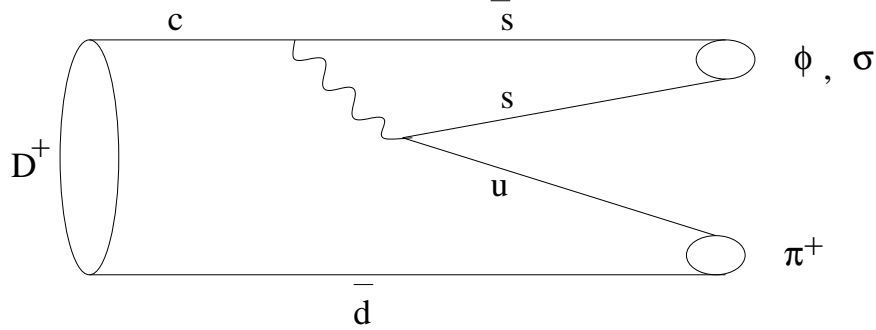


Fig. 1 D^+ decay to π^+ and ϕ or σ via internal W^+ emission

Hamiltonian based on the standard model (see Refs.[14, 15] for reviews of the method). For example, for the decays of the D^+ to a $\phi + \pi^+$ or $\sigma + \pi^+$, which are of central interest for the present work, the effective Hamiltonian is

$$H^{eff} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{qe q_f} [(C_1 (\bar{q}_e q_f)_L (\bar{s} c)_L + C_2 (\bar{s} q_f)_L (\bar{q}_e c)_L], \quad (2)$$

where $(\bar{q}_e q_f)_L = \bar{q}_e^a J_\mu q_f^a \equiv \bar{q}_e^a \gamma^\mu (1 - \gamma_5) q_f^a$, V_{ij} are the CKM matrix elements and the color is summed. The constants C_1, C_2 have been estimated by renormalization group calculations[16]. Calculations with this framework proved to be quite successful for the study of most exclusive D decays. During the period when the method was being developed, and many exclusive charm decays were being measured, however, it was argued that additional flavor singlet gluonic processes, called hairpins, might be important[11, 17]. It was also shown that final state interactions must be considered on the same level as the hairpins[18], and that strong interaction effects at least for certain exclusive decays make it difficult if not impossible to identify the contributions of the various skeletal quark diagrams. For inclusive processes one can carry out operator product expansions and treat the non-perturbative QCD effects using known condensates (see, e.g., Ref [19] for a recent review), however, for exclusive processes accurate inclusion of QCD is difficult.

For the decays $D^+ \rightarrow \phi \pi^+$ and $D^+ \rightarrow \sigma \pi^+$ there is a unique skeletal diagram, the internal W^+ mechanism illustrated in Fig. 1. Using Eq.(3) with a Fierz transformation of the second term one has

$$\langle X \pi^+ | H^{eff} | D^+ \rangle = \frac{G_F}{\sqrt{2}} \cos \theta_c \sin \theta_c C_3 \langle X | \bar{s} s \rangle_L | 0 \rangle \langle \pi^+ | (\bar{u} c)_L | D^+ \rangle. \quad (3)$$

The constant C_3 will not be needed here. The matrix elements of the quarks currents for $X = \phi, \sigma$ are

$$\begin{aligned} \langle 0 | \bar{s} J_\mu s | \phi_\nu \rangle &= f_\phi m_\phi \epsilon_{\mu\nu} \\ \langle 0 | \bar{s} J_\mu s | \sigma \rangle &= f_\sigma m_\sigma \delta_{\mu 0}, \end{aligned} \quad (4)$$

with f_ϕ, f_σ the couplings given by the short-distance values of the wave functions of the ϕ, σ , respectively. For the decay $D^+ \rightarrow \sigma \pi^+$ there is additional contribution from the process with the \bar{s} and s from the two W^+ vertices being replaced by \bar{d} and d . The CKM matrix elements are about the same and $\langle 0 | \bar{d} J_\mu d | \sigma \rangle \simeq 1.2 \langle 0 | \bar{s} J_\mu s | \sigma \rangle$, since the s quark condensate is about 80% of the d quark condensate. Thus the effective value of f_σ is increased by about a factor of two.

Here we use for the value of the ϕ coupling[14] $f_\phi = 228$ Mev. The glueball amplitude constant can be estimated from the low-energy theorem[4]

$$\Pi(0) = \frac{7}{2} \langle 0 | G^2 | 0 \rangle, \quad (5)$$

with $\langle 0 | G^2 | 0 \rangle$ the gluon condensate. The σ coupling can be obtained from another low energy theorem[4]

$$i \int d^4x \langle 0 | T[\bar{s}s(x) \alpha_s G^2(0)] | 0 \rangle \simeq 24\pi \langle 0 | \bar{s}s | 0 \rangle / b_0, \quad (6)$$

with $b_0 = 9.667$ for three colors and two flavors. and Eq.(5), with a similar form for the d -quark contribution to give

$$\langle 0 | \bar{s} J_\mu s | \sigma \rangle + \langle 0 | \bar{d} J_\mu d | \sigma \rangle = 0.14 \delta_{\mu 0} GeV^2. \quad (7)$$

Taking the sigma mass to be 500 Mev, the phase space ratio without spin $\sigma/\phi = 1.23$. Therefore from Eq.(3) we obtain for the ratio of the exclusive decay widths

$$\frac{\Gamma(D^+ \rightarrow \sigma \pi^+)}{\Gamma(D^+ \rightarrow \phi \pi^+)} \simeq 0.15. \quad (8)$$

The experimental value of this ratio from Ref.[1] for the σ and the PDG[20] for the ϕ is 0.22. In the light of both the uncertainties in the calculation and the possibility of other contributions, discussed next, there is quite reasonable agreement between theory and experiment.

Although the diagram shown in Fig.1 is the only skeletal diagram that contributes to either the ϕ or σ decays of the D^+ , there are two other diagram with explicit QCD processes that also contribute: the annihilation-hairpin processes (Fig.2a) and the spectator-final state interaction process (Fig. 2b). The later is analogous to the process treated in Ref.[18] in which it was shown that such final state rearrangement processes might be comparable in magnitude to the nonspectator decays. We do not attempt to estimate these processes which are very difficult to calculate accurately.

For the D_s^+ decay into a $\phi \pi^+$ there is one skeletal diagram, the spectator shown in Fig.3a; while for the decay into a $\sigma \pi^+$ there is a contribution from the spectator graph and also two annihilation graph processes, shown in Fig.3b. For the $D_s^+ \rightarrow \sigma \pi^+$ the matrix element of the weak effective Hamiltonian, after a Fierz transformation, is

$$\langle \sigma \pi^+ | H^{eff} | D_s^+ \rangle = \frac{G_F}{\sqrt{2}} \cos^2 \theta_c (C_1 + 3C_2) \langle \sigma \pi^+ | (\bar{u}d)_L | 0 \rangle \langle 0 | (\bar{s}c)_L | D_s^+ \rangle \quad (9)$$

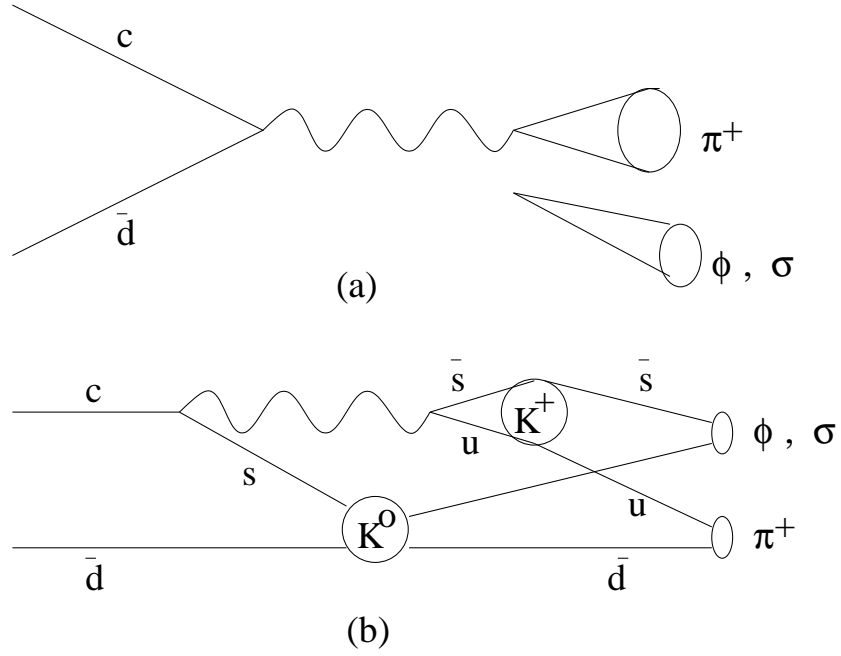


Fig. 2 (a) Annihilation-hairpin, (b) Final state interaction

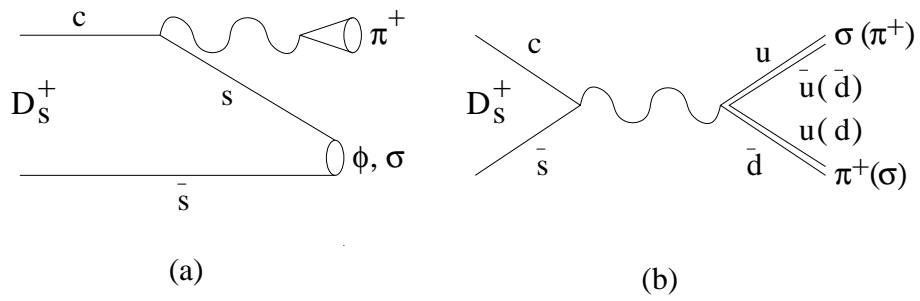


Fig. 3 (a) Spectator, (b) Annihilation

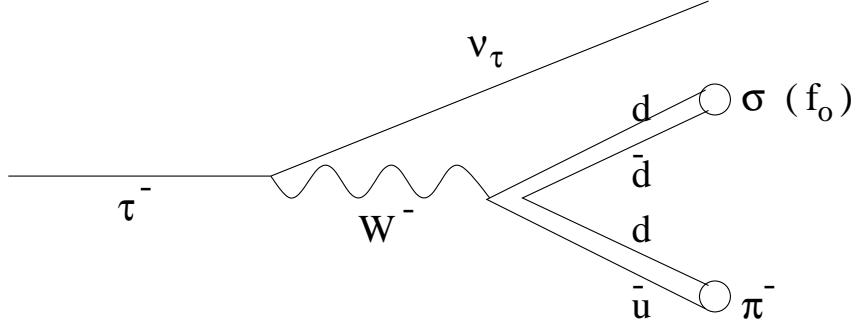


Fig. 4 Diagram for $\tau^- \rightarrow \nu_\tau \pi^- \sigma$ (or scalar meson)

Using the results given in Ref.[14] one sees that the ratio of the σ to ϕ rates for the D_s^+ decays, which contains a factor of $(C_1 + 3C_2)^2/C_1^2$, would be strongly reduced compared to the D^+ decay. This gives a qualitative explanation for the fact that the $\sigma\pi^+$ channel was not found[3] in the E791 experiment.

It is interesting to note that the only skeletal diagram for the $D^+ \rightarrow K^+ + \phi(\sigma)$ decays is the annihilation process; and that there are also two D^0 decays to the σ or ϕ , $D^0 \rightarrow \phi(\sigma)\bar{K}^0$ and $D^0 \rightarrow \phi(\sigma)K^0$, that have the W -exchange as the only skeletal diagram. We expect that the ratio of the σ to the ϕ fraction should be similar to the corresponding D^+ decays described above. Another interesting related experimental observation by the CLEO Collaboration[21] is that in the $\tau^- \rightarrow \nu_\tau \pi^- \pi^0 \pi^0$ decay the data cannot be fit without a sigma channel. The diagram with the W^- coupling to the $\sigma\pi^-$ through a $d\bar{u}$, illustrated in Fig.4, leads to this channel in a natural way in the glueball/sigma model. In this case a more detailed theoretical study of the $(\bar{q}_a q_b)_L$ matrix elements is needed to obtain the branching ratios, which we do not attempt here.

We conclude that the D^+ and D_s decays are consistent with our glueball/sigma model and suggest that experimental searches for the sigma channels in other charm decays, as well as other heavy quark and lepton hadronic decays, would be rewarding.

[†] On leave from Carnegie Mellon University

This work was supported in part by the NSF grant PHY-00070888. The author wishes to thank the TQHN group at the University of Maryland for hospitality during the period when this work was carried out, and acknowledge helpful discussions with Jen-chieh Peng, Roy Briere, Jeffrey Appel and Thomas Cohen.

References

- [1] E791 Collaboration, E.M. Aitala *et.al.*, Phys. Rev. Lett. **86**, 770 (2001).
- [2] B.S. Zou and D.V. Bugg, Phys. Rev. **D50**,591 (1995).

- [3] E791 Collaboration, E.M. Aitala *et.al.*, Phys. Rev. Lett. **86**, 765 (2001).
- [4] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. **B 165** (1980) 67; **B 191**, 301 (1981).
- [5] C.A. Dominguez and N. Paver, Zeit. Phys. **C 31**, 591 (1986); J. Bordes, V. Gimenez and J.A. Penarrocha, Phys. Lett **B 223**, 251 (1989); J. Liu and D. Liu, J. Phys. G:Nucl. Part. Phys. **19**, 373 (1993); E. Bagan and T.G. Steele, Phys. Lett. **B243**, 413 (1990); L.S. Kisslinger, J. Gardner and C. Vanderstraeten, Phys. Lett.**B410**, 1 (1999); H. Forkel, hep-ph/0005004 (2000).
- [6] L.S. Kisslinger and M.B. Johnson, in preparation.
- [7] Crystal Barrel Collaboration, Phys. Lett. **B 355** (1995) 425.
- [8] Y. Zhu, BES J/ψ group, Proceedings of the 6th Conference on the Intersection of Particle and Nuclear Physics, Big Sky Montana (T.W. Donnelly, Ed., 1997), p. 476.
- [9] L.S. Kisslinger and Z. Li, Phys. Lett. **B445**, 271 (1999).
- [10] L.S. Kisslinger and W-h Ma, Phys. Lett. **B485**, 367 (2000);. L.S. Kisslinger, W-h Ma and P. Shen, hep-ph/0005052.
- [11] L.L. Chau, Phys. Rep. 95, 1 (1983).
- [12] M. Bander, D. Silverman and A. Soni, Phys. Rev. Lett **44**, 7 (1980); H. Fritzsch and P. Minkowski, Phys. Lett. **90B**, 455 (1980); K. Shizuya, Phys. Lett. **100B**, 79 (1981).
- [13] L.L. Chau and H-Y. Cheng, Phys. Rev. **D36**, 137 (1987);
- [14] A.J. Buras, J.M. Gérard and R. Rückl, Nucl. Phys. **B268**, 16 (1986)
- [15] M. Bauer, B. Stech and M. Wirbel, Z. Phys. **C34**, 103 (1987).
- [16] M.K. Gaillard and B.W. Lee, Phys. Rev Lett. **33**, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. **52B**, 351 (1974); J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. **100**, 313 (1975).
- [17] X.Y. Li and X.Q. Li, Nuovo Cimento **100A**, 693 (1988); Chau and H-Y. Cheng, Phys. Rev. **D39**, 2788 (1989).
- [18] J.F. Donoghue, Phys. Rev. **D36**, 1516 (1986).
- [19] I. Bigi and N. Uraltsev, Phys. Rev. **60**, 114034 (1999).
- [20] Particle Data Group, C Caso *et.al.*, European Phys. Jour. **C3**, 1 (1990).
- [21] Cleo Collaboration, D. Asner *et.al.*, Phys. Rev. **D61**, 012002 (2000).